

The Soft Substructure of Jets

Duff Neill

MIT

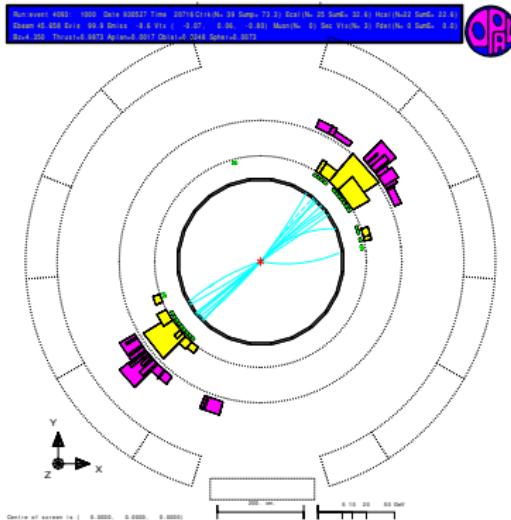
Fermilab 2016

March 10, 2016

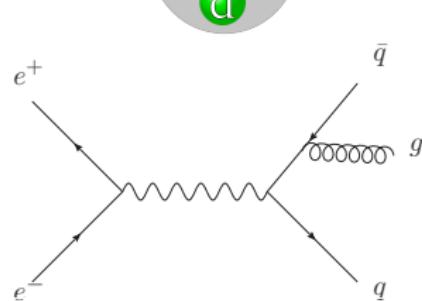
Quantum Chromodynamics (QCD)

Quantum Chromodynamics:

- High energy, quasi-conformal, weakly coupled & massless (asymptotic freedom).
 - Low energies, strongly confines into bound states.



Dijet event in OPAL detector at LEP.



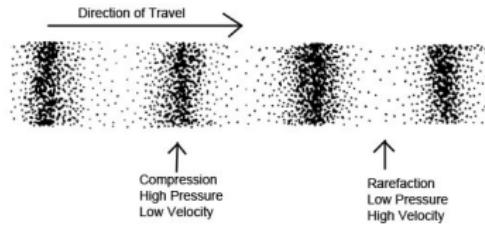
The Basic Questions:

- How does an event at high energy transition from interacting partons to strongly bound non-interacting hadrons?
- How does the reverse happen?
- How does QCD quantum mechanically transfer momentum, spin, charge, flavor, etc., between the ultra-violet and the infra-red?

Factorization and Effective Field Theory

- Separation of scales and universality of infra-red or long-distances.
- Form is dictated by symmetries.
- c_s is the ultra-violet or microscopic physics.

$$\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$



Building the Tool-set To Crack Open the QCD Shower

- Formulate calculable *observables* that track the momentum flow.
- Derive factorization formulas to organize the calculation.
- Understand the limits of factorization.
- Characterize the organization of the states of QCD at each scale.

Outline

- Factorization and Soft Collinear Effective Field Theory (SCET).
- Defining a Jet: The Broadening Axis
[Larkoski, **DN**, Thaler **1401.2158**].
- Discriminating Jets: Jets within Jets
[Larkoski, Moult, **DN** **1409.6298**, **1501.04596**, **1507.03018**].
- Calculating with Jets: Integrating the Histories of QCD Showers
[Larkoski, Moult, **DN** **1501.04596**], [**DN** **1508.07568**].

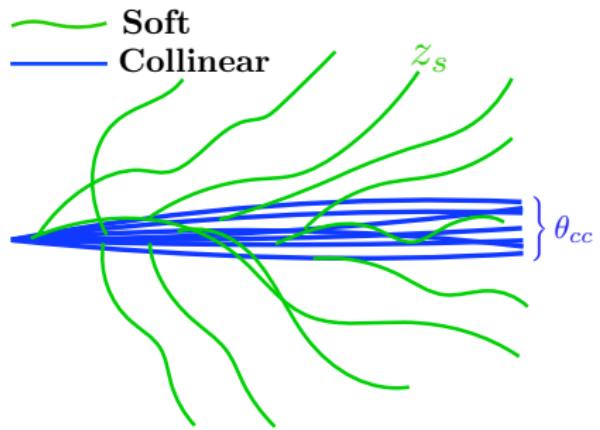
The “states” of Perturbation Theory

- Quarks and gluons are massless when on-shell: $p_g^2 = p_q^2 = 0$.
- High energies, no intrinsic scale, except hard interaction.

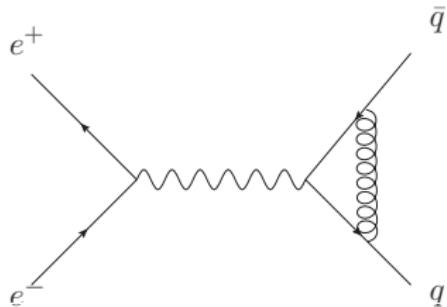
$$m_{ab}^2 = (p_a + p_b)^2 = 4E_a E_b \sin^2 \frac{\theta_{ab}}{2} \sim Q^2 z_a z_b \theta_{ab}^2$$

Close to on-shell:

- Soft:
 $\left(\sum p_s \right)^2 \sim Q^2 z_s^2 \ll Q^2$
- Collinear:
 $\left(\sum p_c \right)^2 \sim Q^2 \theta_{cc}^2 \ll Q^2$

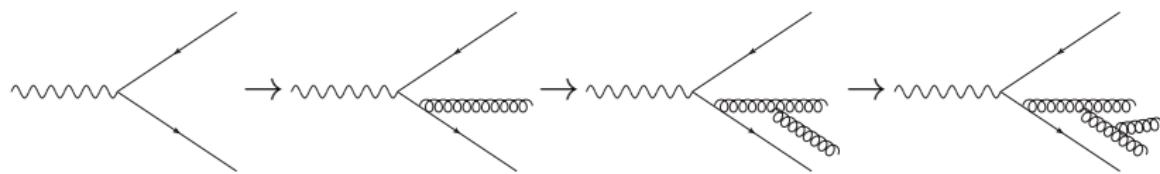


The Soft and Collinear Contributions To Perturbation Theory



- Feynman Diagrams for the process exhibit singularities in these soft and collinear regions.
- Singularities correspond to physically realizable processes.
- Enhancement of singular regions → formation of jets.

The Soft and The Collinear: Cascade of a Jet



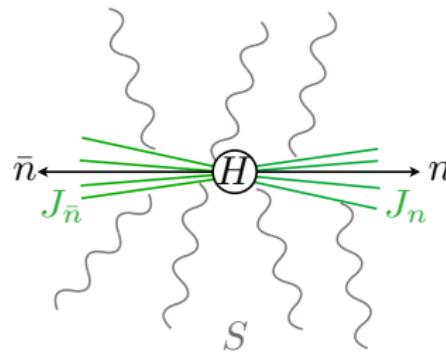
- Perturbative Problem: What are the observed states of a theory where long-ranged interactions do not turn off?
- Non-Perturbative Problem: How does the cascade physically end?

Qualitative cure: Infra-red safe observables that are insensitive to the full shower history.

Soft-Collinear Effective Field Theory

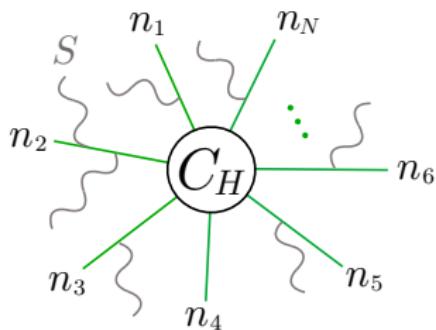
SCET, a grammar for factorization:

- Operators defined by the IR limits of QCD.
- Constrained by symmetries of QCD.
- Constructs a systematic expansion of QCD observables.
- All momentum transfers between sectors expanded according to the power counting.
- Renormalization Scales and Subtractions to distinguish modes.



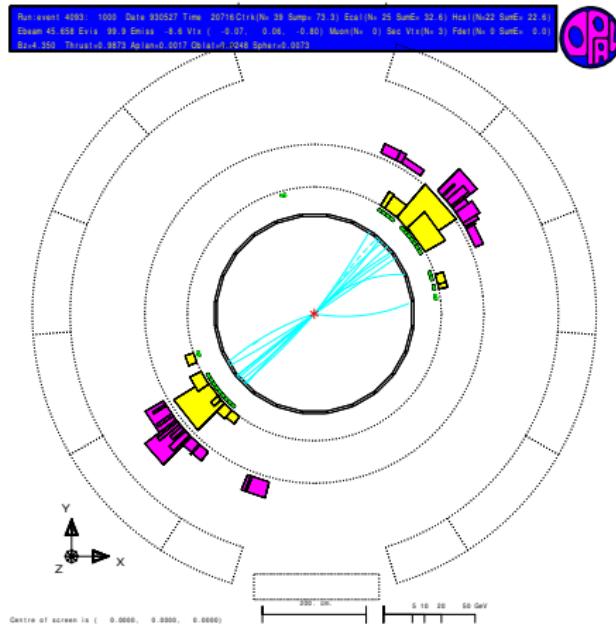
Factorization Theorem

$$\frac{d\sigma}{d\tau} = \text{tr}[C_H(Q^2, \mu)S(\tau, \mu)C_H^\dagger(Q^2, \mu)] \otimes J_{n_1}(\tau, \mu) \otimes J_{n_2}(\tau, \mu) \otimes \dots + O\left(\frac{\tau^2}{Q^2}\right)\dots$$



Rules of SCET facilitate the derivation of the factorized form.

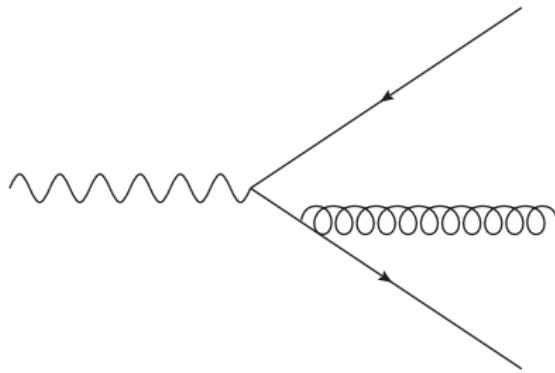
Defining a Jet



No intrinsic scale to QCD jets.

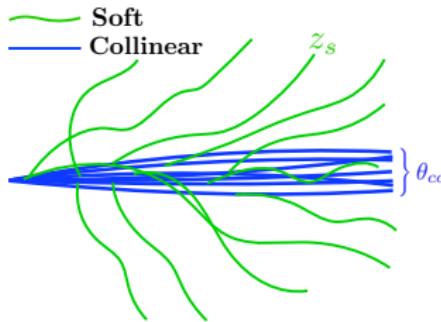
- Issues: Direction and Energy Flow.
- Goal: vertex back to hard interaction.

Starting a Jet



Setting direction and energy flow: Initial Splittings of QCD.

Thrust and Broadness of a Jet



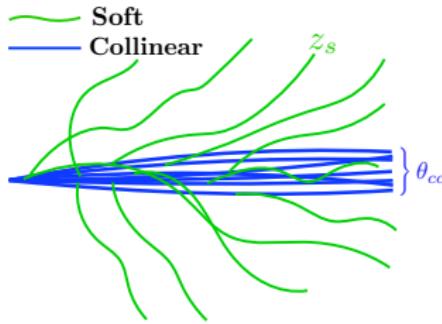
- Average of radiation in a region: "Thrust axis"
- Demand transverse momenta are small: "broadening".

$$\hat{t} = \frac{1}{Q} \sum_{i \in J} \vec{p}_i$$

$$\tau_t^{(1)} = \frac{1}{Q} \sum_{i \in J} |\vec{p}_{i\perp \hat{t}}| \sim \sum_{i \in J} z_i \theta_{i\hat{t}}$$

Broadening is an event-shape.

Broadening Power Counting



Power counting the observable:

$$\tau_t^{(1)} \sim \sum_{i \in J} z_i \theta_{i\hat{t}} \ll 1$$

$$\text{Soft :} \left(\sum p_s \right)^2 \sim (Q z_s)^2 \sim (Q \tau_t^{(1)})^2$$

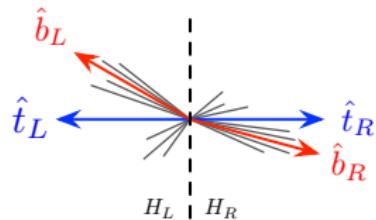
$$\text{Collinear :} \left(\sum p_c \right)^2 \sim (Q \theta_{cc})^2 \sim (Q \tau_t^{(1)})^2$$

SCET_{II} [Chiu,Jain,**DN**,Rothstein **1202.0814**]

Issue with the Thrust Axis: Recoil

- Goal: vertex back to hard interaction.
- Thrust axis sensitive to outliers and perturbations.
- Broadening Axis minimizes broadening.
- Broadening Axis: a multi-dimensional generalization of the median

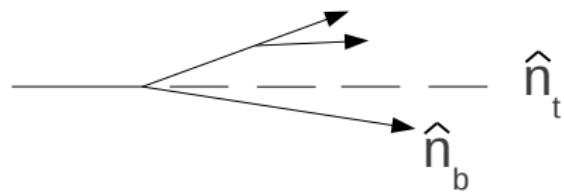
$$\tau_b^{(1)} = \frac{1}{Q} \min_{\hat{b}} \sum_{i \in J} |\vec{p}_{i \perp \hat{b}}|$$



Naturally aligns with cluster of hardest radiation.
[Larkoski, **DN**, Thaler 1401.2158]

Issue with the Thrust Axis: Recoil

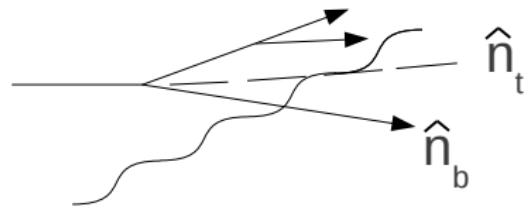
- Goal: vertex back to hard interaction.



Naturally aligns with cluster of hardest radiation.

Issue with the Thrust Axis: Recoil

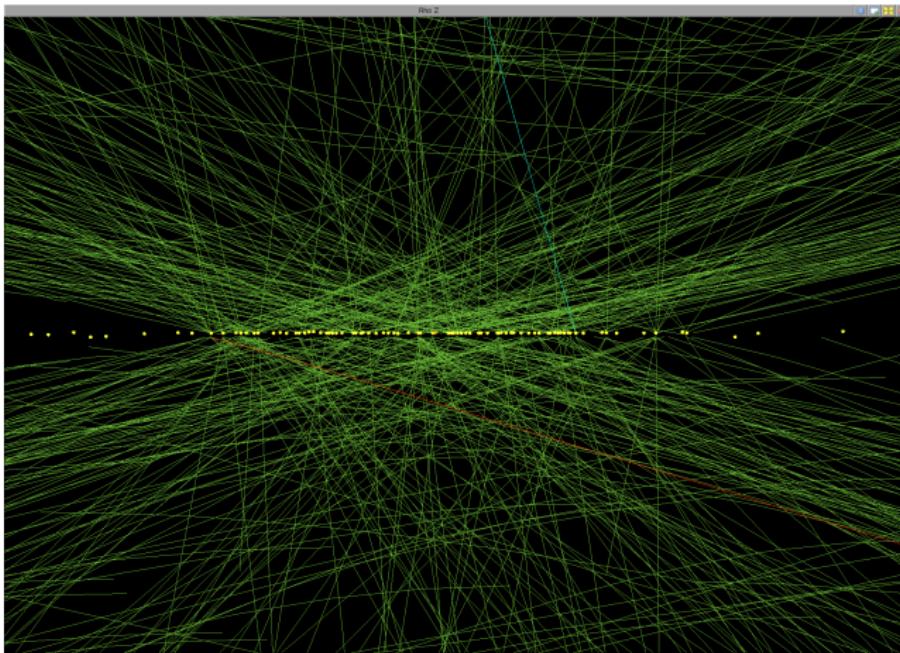
- Goal: vertex back to hard interaction.



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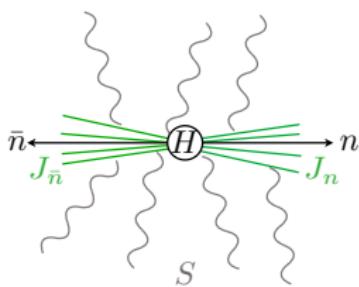
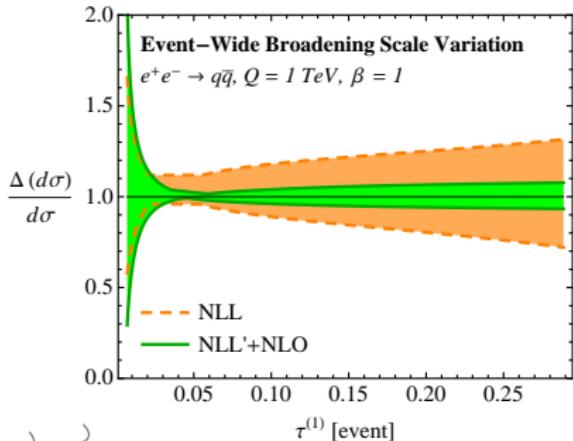
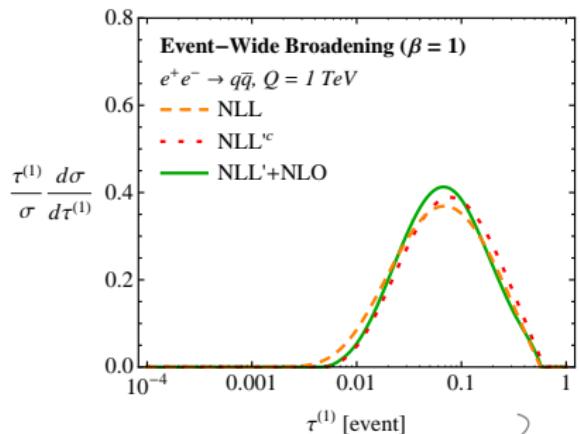
Issue with Recoil at LHC

78 Simultaneous Events at CMS detector at 8TeV.



Jet Spectra

$$\frac{d\sigma}{d\tau^{(1)}} = H(Q^2, \mu) J_n(\tau^{(1)}, \mu, \nu) \otimes J_{\bar{n}}(\tau^{(1)}, \mu, \nu) \otimes S_{n\bar{n}}(\tau^{(1)}, \mu, \nu)$$



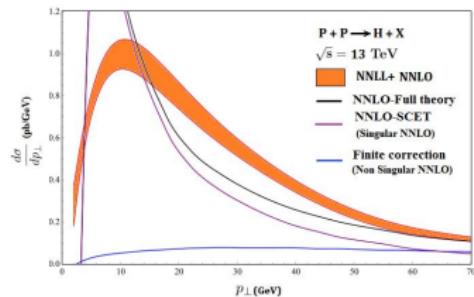
Rapidity Factorization

Broadening belongs to SCET_{II} theories.

Generalized Rapidity Renormalization

[Chiu,Jain,**DN**,Rothstein **1202.0814**]

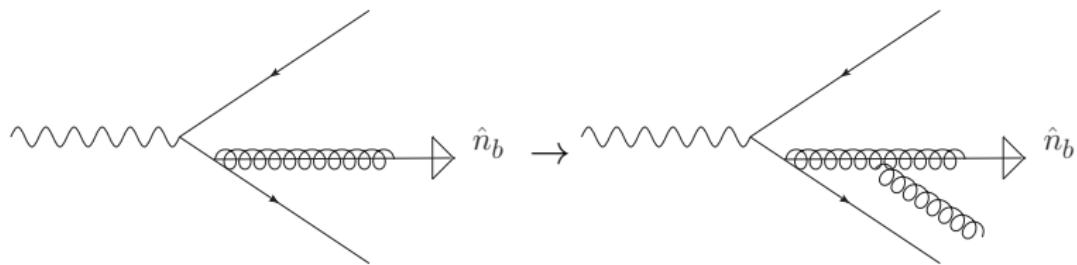
- P_T vetos for Higgs Searches
- Transverse Momentum Spectra
- BFKL
- Exclusive B -meson decays



[**DN**,Rothstein,Vaidya
1503.00005]

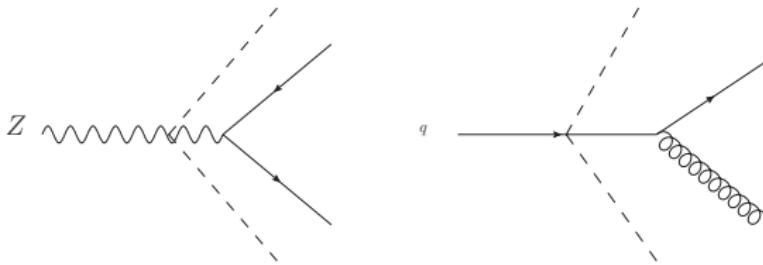
Broadening Axis And Bulk Jet Properties

- Broadening Axis gives jet direction robustly.
- Insensitive to the subsequent Soft History of the Jet



Jets Within Jets

- Having vertexed back to hard interaction point, what was the interaction?

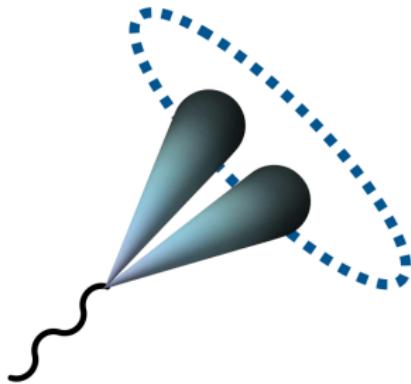
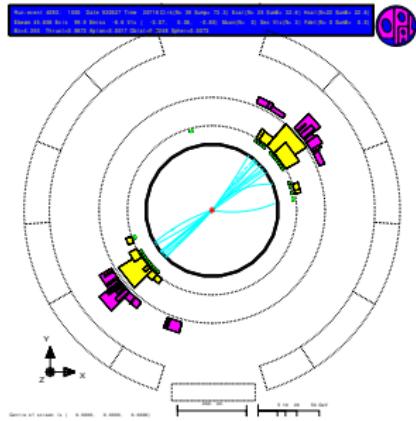


- Bulk energy flow will not discriminate.
- Pry open the cascade history of the jet.

[Larkoski, Moult, **DN 1409.6298,1501.04596,1507.03018**]

Jets Within Jets

In rest frame, color singlet objects decay back-to-back:



Boosted, will still exhibit two pronged decays within a fatter jet.

Color Decaying Resonances Versus QCD

Z-jets: Mass and energy gives natural opening angle:

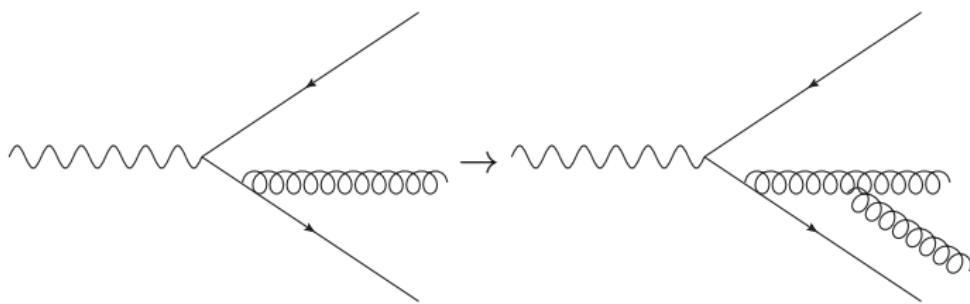
$$\frac{M_Z^2}{Q^2} = \frac{(p_q + p_{\bar{q}})^2}{Q^2} \sim R^2$$

QCD-jets: No Characteristic mass or opening angle, no preference for structure, populate all regions.

Measure Two pronged-ness

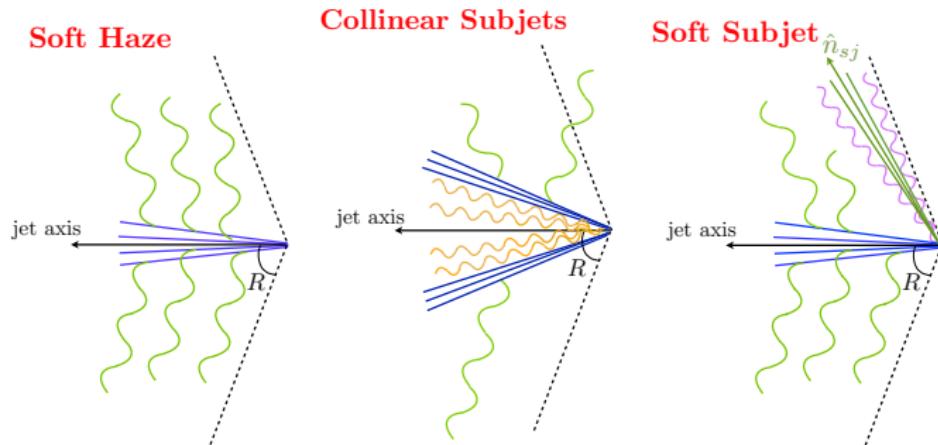
Idea: find jets with internal structure

- At Z -mass scale.
- Veto hazy structure.



Ordering the Shower: The QCD question

When are subsequent emissions at the same scale or strongly ordered?



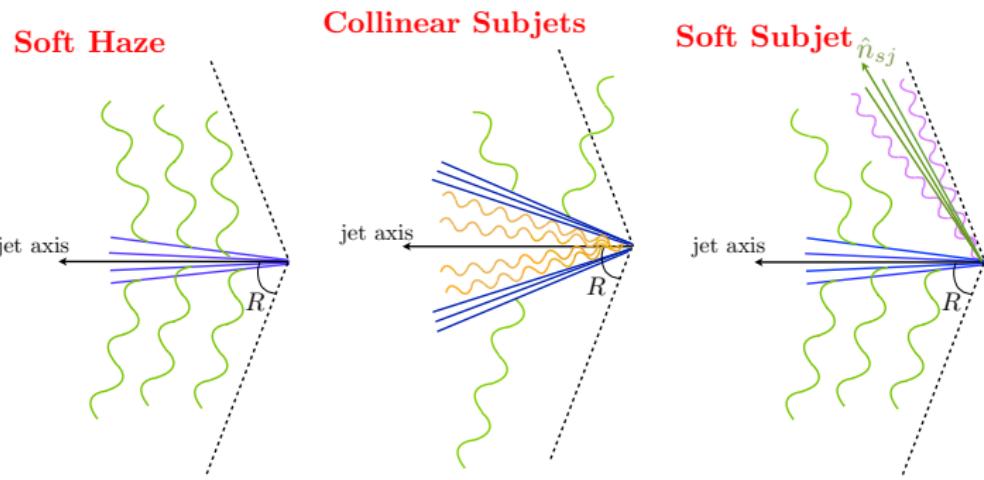
- There exists a general organization of the phase-space, applicable to many observables.
- Parametrically separate distinct regions.
- Bulk energy flow can be identical.

[Larkoski, Moult, **DN 1409.6298**]

Energy Correlation Functions

$$e_2^{(\alpha)} = \sum_{i < j \in J} z_i z_j \theta_{ij}^{\alpha}$$

$$e_3^{(\alpha)} = \sum_{i < j < k \in J} z_i z_j z_k \left(\theta_{ij} \theta_{jk} \theta_{ki} \right)^{\alpha}$$



Energy Correlation Functions: One Prong

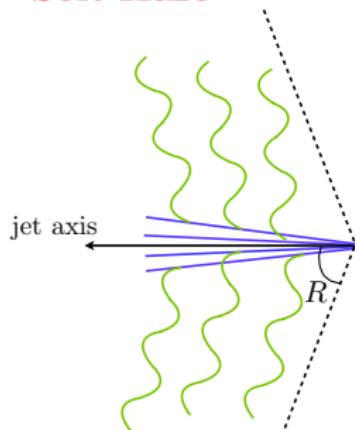
$$e_2^{(\alpha)} = \sum_{i < j \in J} z_i z_j \theta_{ij}^{\alpha} \ll 1$$

Power Counting:

$$z_s \sim e_2^{(\alpha)}$$

$$\theta_{cc}^{\alpha} \sim e_2^{(\alpha)}$$

Soft Haze



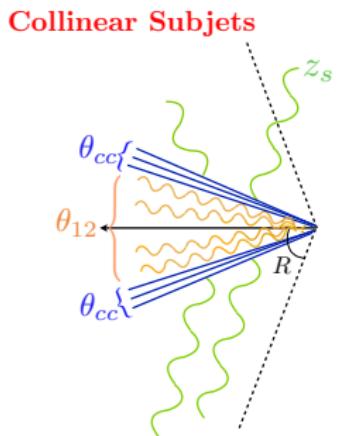
Correlate Two Soft and One Collinear:

$$e_3^{(\alpha)} \sim (z_c z_s^2 \theta_{sc}^{2\alpha} \theta_{ss}^{\alpha}) \sim (e_2^{(\alpha)})^2$$

Energy Correlation Functions: Two Prongs

$$e_2^{(\alpha)} = \sum_{i < j \in J} z_i z_j \theta_{ij}^\alpha \sim z_1 z_2 \theta_{12}^\alpha + \dots$$

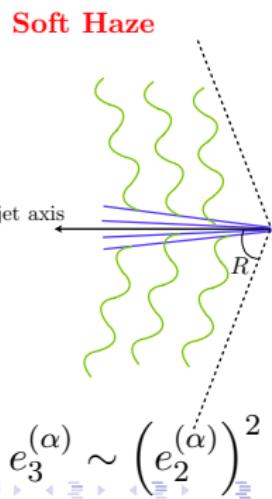
$$e_3^{(\alpha)} = \sum_{i < j < k \in J} z_i z_j z_k (\theta_{ij} \theta_{jk} \theta_{ki})^\alpha \sim z_1 z_2 \theta_{12}^\alpha (\theta_{12}^\alpha \theta_{cc}^\alpha + z_s) \dots$$



$$z_s \ll e_2^{(\alpha)} \text{ or } \theta_{cc} \ll e_2^{(\alpha)}$$

Soft haze and subjet regions
parametrically
separated if

$$e_3^{(\alpha)} \sim (e_2^{(\alpha)})^3$$



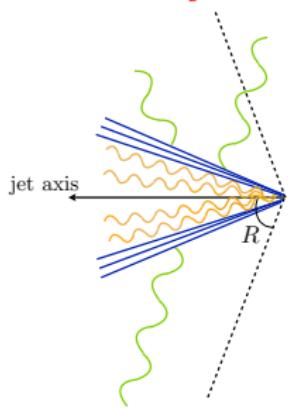
$$e_3^{(\alpha)} \sim (e_2^{(\alpha)})^2$$

Collinear Versus Soft Subjets

$$e_2^{(\alpha)} \rightarrow z_1 z_2 \theta_{12}^\alpha + \dots$$

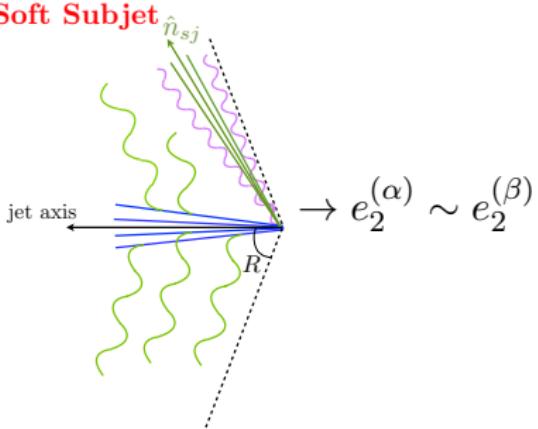
$$e_2^{(\beta)} \rightarrow z_1 z_2 \theta_{12}^\beta + \dots$$

Collinear Subjets

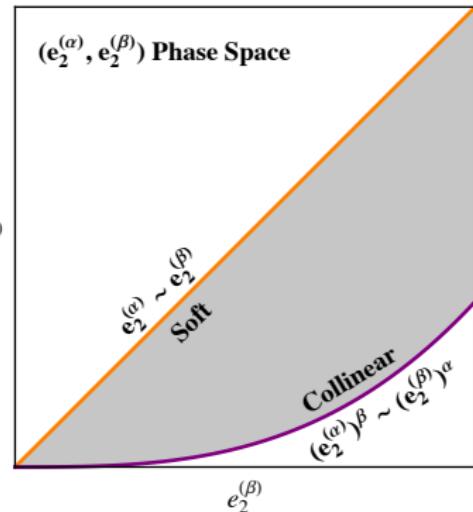
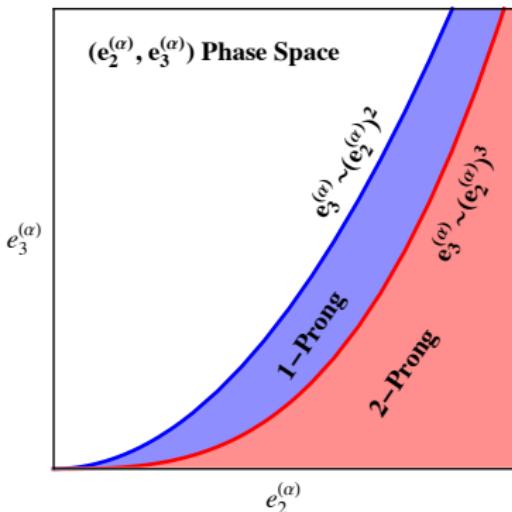


$$\rightarrow e_2^{(\alpha)} \sim \left(e_2^{(\beta)} \right)^{\frac{\alpha}{\beta}}$$

Soft Subjet



Phase Space for $1 \rightarrow 2$ Splitting

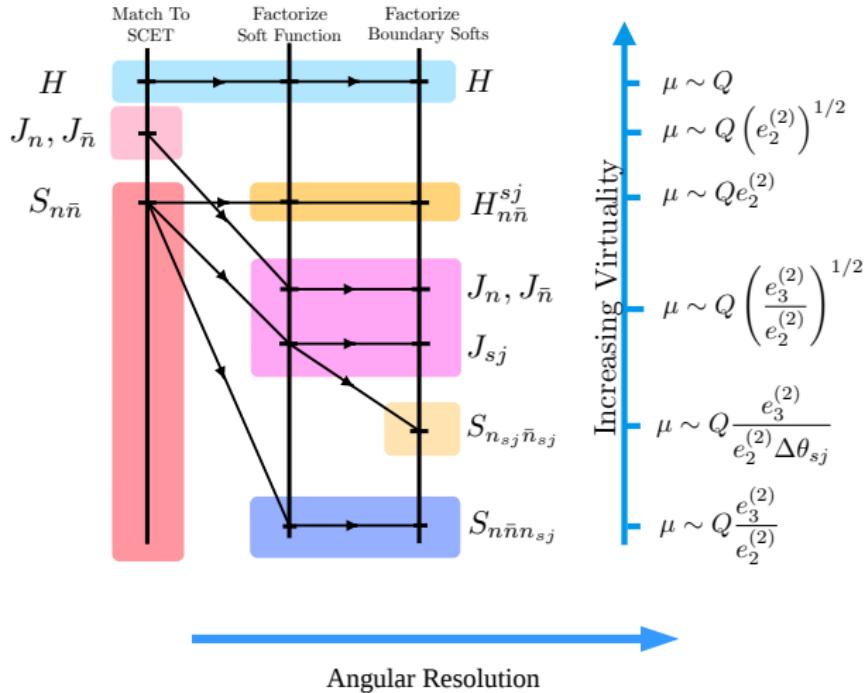


Single observable distinguishes 1-prong (**background**) from the 2-prong (**signal**) regions:

$$D_2 = \frac{e_3^{(\alpha)}}{\left(e_2^{(\alpha)}\right)^3}$$

Soft Subject Factorization in SCET

$e^+e^- \rightarrow 2_j + 1_{sj}$:



[Larkoski, Moult, **DN 1501.04596**]

Soft Subjet Factorization Theorem

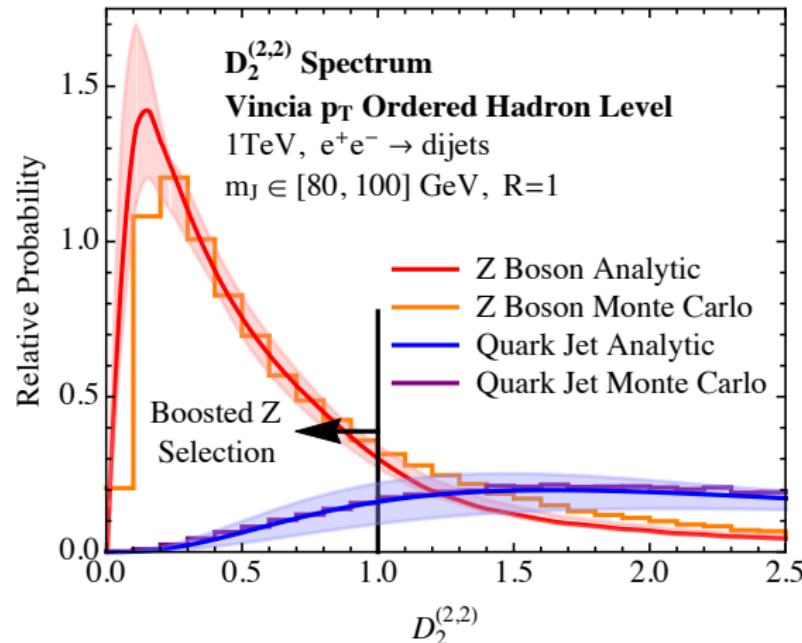
$e^+e^- \rightarrow 2_j + 1_{sj}$:

$$\frac{d\sigma}{de_2^{(\alpha)} de_2^{(\beta)} de_3^{(\beta)} d\tau_B} = H_{n\bar{n}} H_{n\bar{n}}^{sj}(e_2^{(\alpha)}, e_2^{(\beta)}) J_{n_{sj}}(e_3^{(\beta)}) \otimes S_{n_{sj}\bar{n}_{sj}}(e_3^{(\beta)}) \\ \otimes S_{n\bar{n}n_{sj}}(e_3^{(\beta)}; \tau_B) \otimes J_n(e_3^{(\beta)}) \otimes J_{\bar{n}}(\tau_B)$$

- Each function encodes the dynamics of different contributing phase space regions.

NLL Analytic Predictions For Jet Substructure

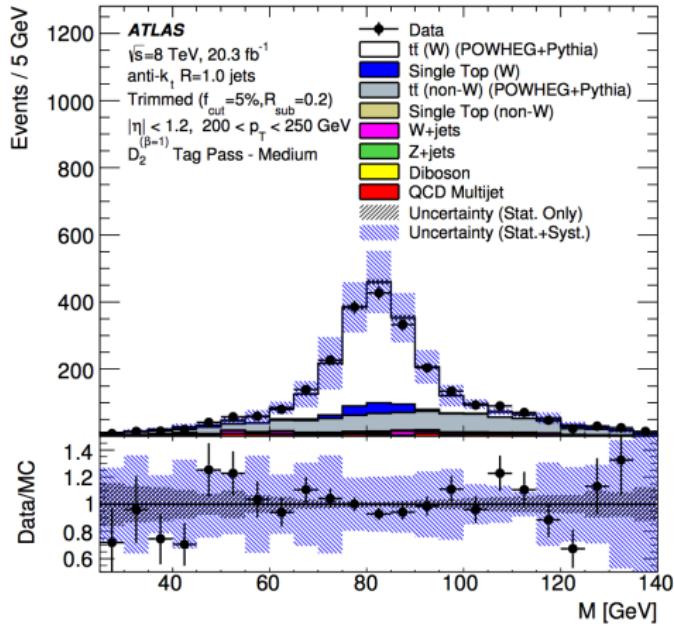
Combine Soft and Collinear Subjets Factorizations:



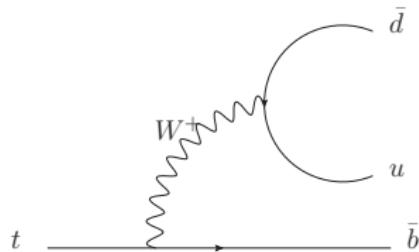
First analytic calculation for both signal and background in jet substructure. [Larkoski, Moult, [DN 1507.03018](#)]

Experimental Results

ATLAS [1510.0582] Measurement of Hadronic Decaying W Mass, using D_2 for tagging:

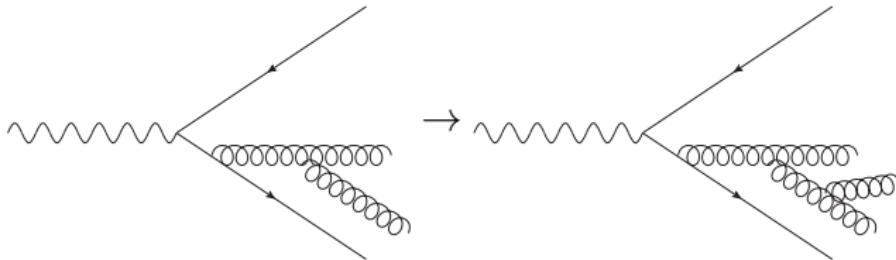


Deeper into the Cascade: Discriminating Top Quarks



Need resolution scale one splitting lower:

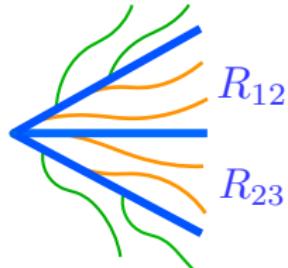
$$e_4^{(\alpha)} = \sum_{i < j < k < \ell \in J} z_i z_j z_k z_\ell \left(\theta_{ij} \theta_{jk} \theta_{kl} \theta_{li} \theta_{lj} \theta_{kj} \theta_{ik} \right)^\alpha$$



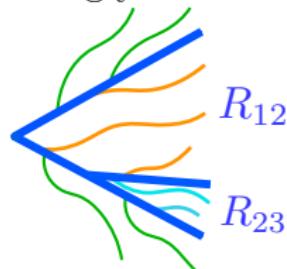
[Larkoski, Moult, **DN 1411.0665**]

Varieties of $1 \rightarrow 3$ Splittings

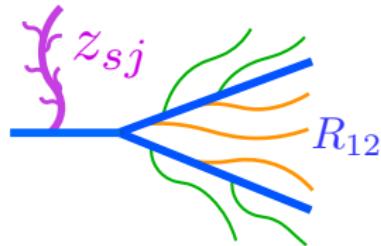
Unordered Subjets:



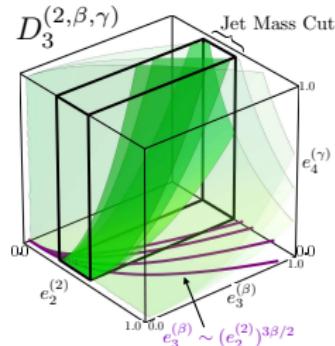
Strongly Ordered:



Soft & Collinear Subjets:

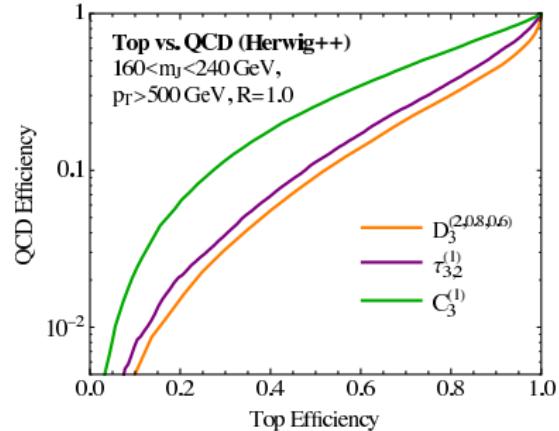
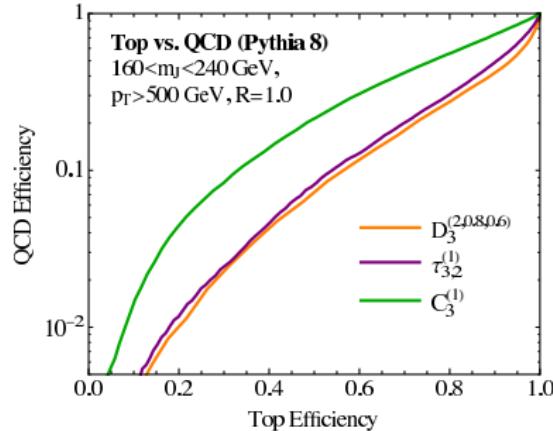


R is opening angle.



Power Counting Gives Discriminator

Comparison of Discriminators At 8 TeV LHC:



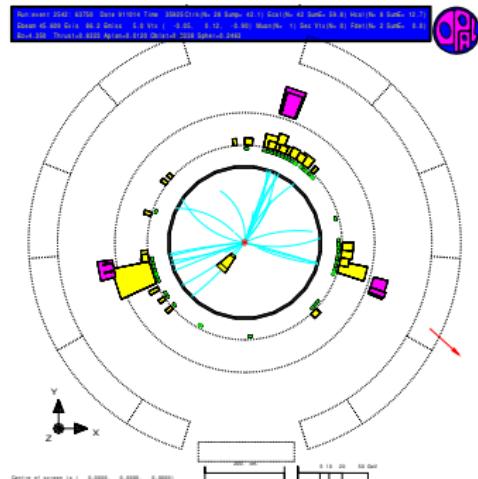
$$D_3^{\alpha,\beta,\gamma} = e_4^{(\gamma)} \left(\frac{\left(e_2^{(\alpha)}\right)^{\frac{3\gamma}{\alpha}}}{\left(e_3^{(\beta)}\right)^{\frac{3\gamma}{\beta}}} + x \frac{\left(e_2^{(\alpha)}\right)^{\frac{2\gamma}{\alpha}-1}}{\left(e_3^{(\beta)}\right)^{\frac{2\gamma}{\beta}}} + y \frac{\left(e_2^{(\alpha)}\right)^{\frac{2\beta}{\alpha}-\frac{\gamma}{\alpha}}}{\left(e_3^{(\beta)}\right)^2} \right)$$
$$x \sim \left(\frac{(p_T^{cut})^2}{m_{top}^2} \right)^{\frac{\alpha\gamma}{\beta}-\frac{\alpha}{2}}, \quad y \sim \left(\frac{(p_T^{cut})^2}{m_{top}^2} \right)^{\frac{5\gamma}{2}-2\beta}$$

Working back up the Cascade

- Studied more and more exclusive cross-sections.
- Can this reveal information about more inclusive distributions?

Correlations Throughout the Event

- **Pictures** treated jets as evolving but isolated states.
 - Actually embedded into larger event.
 - Encoded in the **soft factor** of the Factorization Theorem.

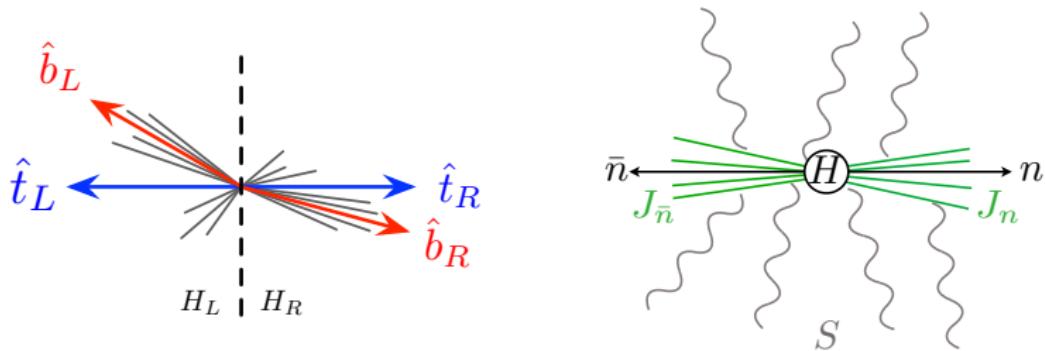


The Two-Jet Factorization

Two Jet Factorization For Broadening in Two Hemispheres:

$$\frac{d\sigma}{d\tau_L^{(1)} d\tau_R^{(1)}} = H(Q^2, \mu) J_n(\tau_L^{(1)}, \mu, \nu) \otimes J_{\bar{n}}(\tau_R^{(1)}, \mu, \nu) \otimes S_{n\bar{n}}(\tau_L^{(1)}, \tau_R^{(1)}, \mu, \nu)$$

$$\tau_R^{(1)} < \tau_L^{(1)} \ll 1$$

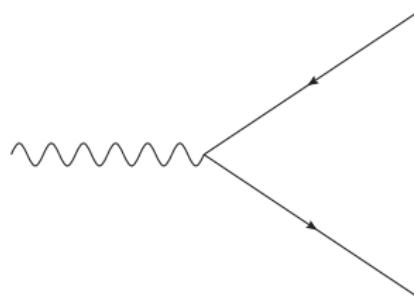


Cascading the Broader Jet

Since:

$$\tau_L^{(1)} > \tau_R^{(1)}$$

There is “time” for broader jet to develop sub-jets:



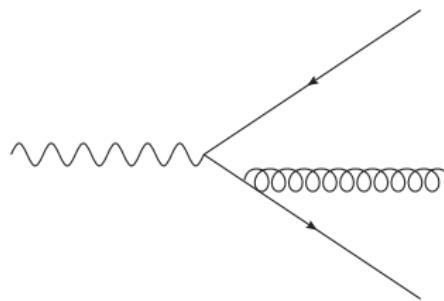
Before the narrower jet splits.

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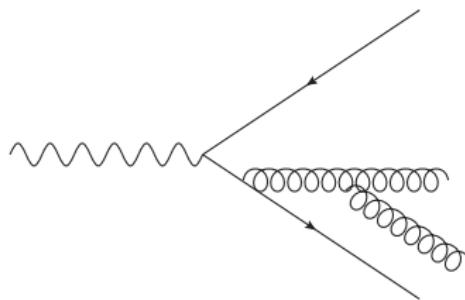
Before the narrower jet splits.

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There is “time” for broader jet to develop sub-jets:



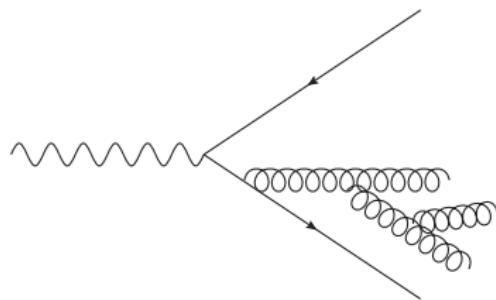
Before the narrower jet splits.

Cascading the Broader Jet

Since:

$$\tau_L^{(1)} > \tau_R^{(1)}$$

There is “time” for broader jet to develop sub-jets:



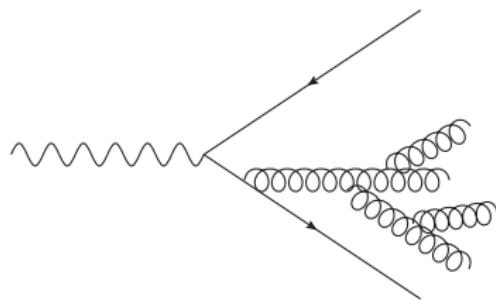
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Cascading the Broader Jet

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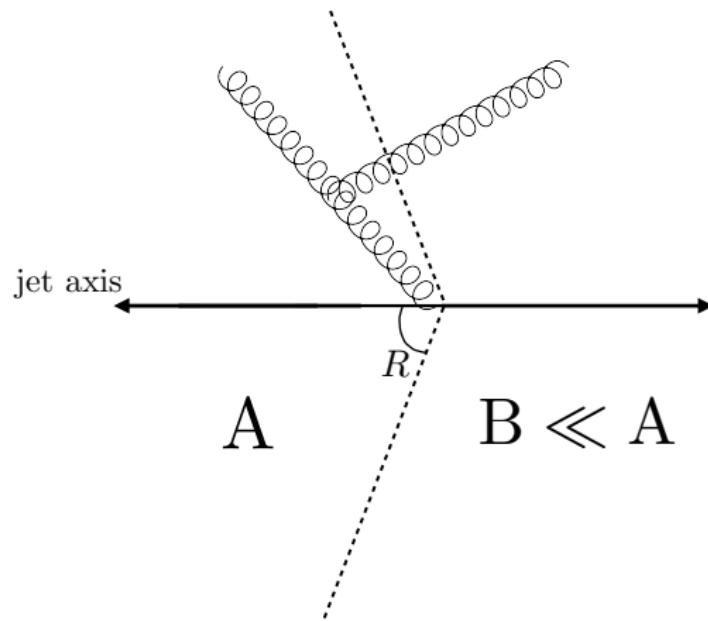
There is “time” for broader jet to develop sub-jets:



Before the narrower jet splits.

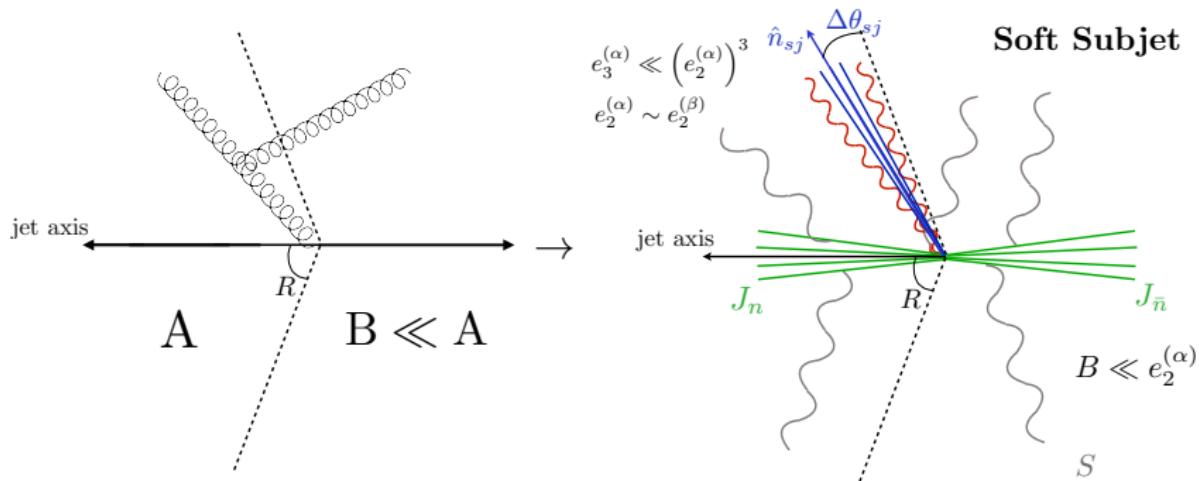
Secondary Branching Between Jets

- Soft Radiation Entangles the Measurements of the Jets.
- Relative Distribution of Jets:
Manifested in the *Non-Global Logarithm* (NGL) $\ln \frac{\tau_L}{\tau_R}$.



Jet Substructure Understanding of Secondary Branching

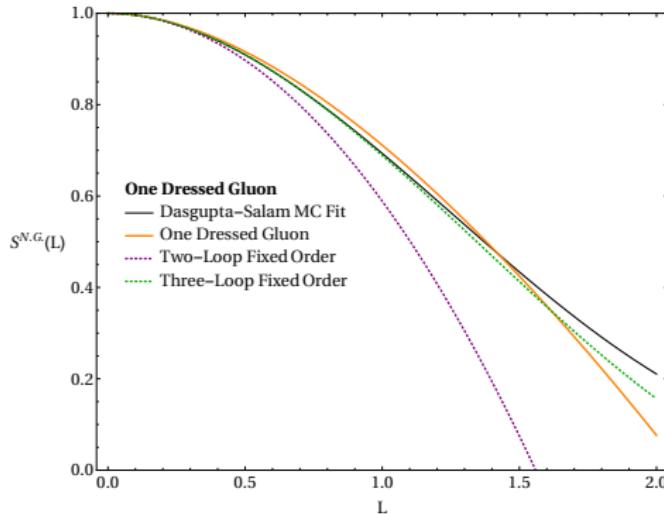
- Jet Substructure: Applying sub-jet factorizations to reorganize NGLs.



[Larkoski, Moult, **DN 1501.04596**
[**DN 1508.07568**]

Soft Jet Contribution to More Inclusive Factorization

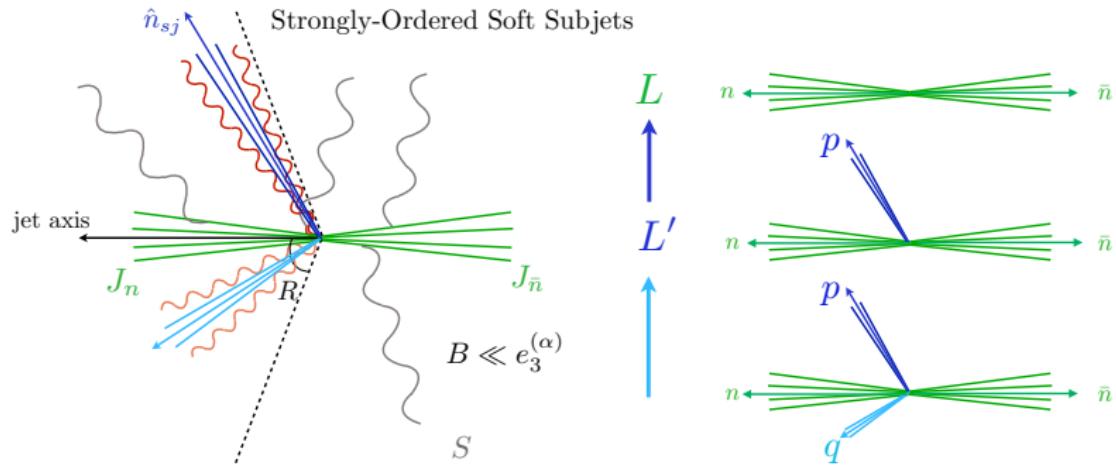
Factorization Dresses Gluon With Non-Trivial Resummation.



$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{\tau_L}{\tau_R}$$

A fixed order expansion using resummed propagators.

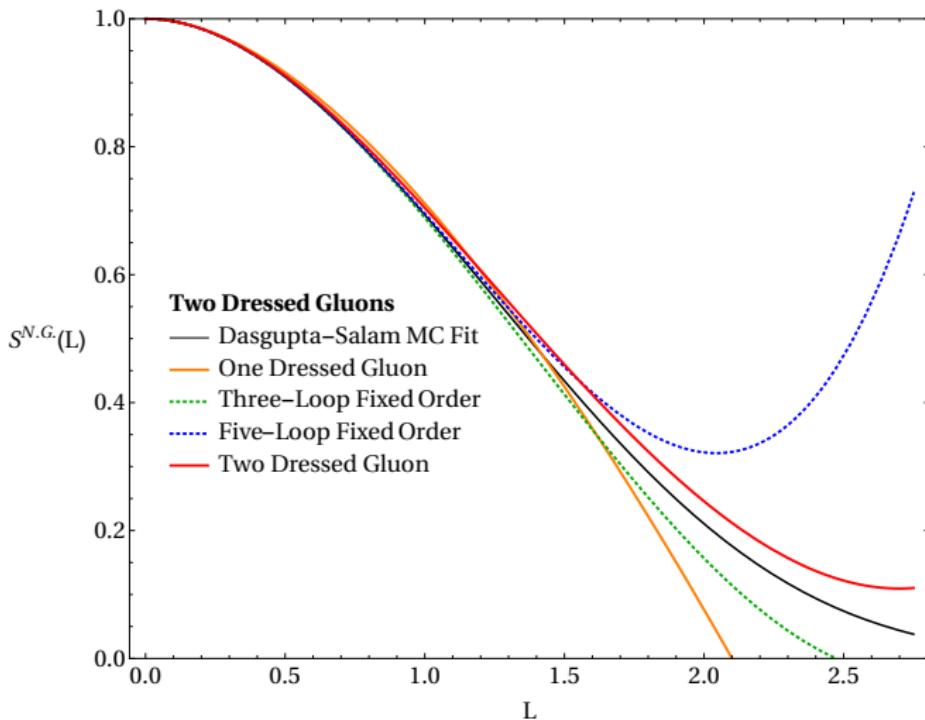
Iterating Soft jet factorization.



Sequence of EFTs to improve NGL resummation:

$$0_{sj} \rightarrow 1_{sj} \rightarrow 2_{sj}$$

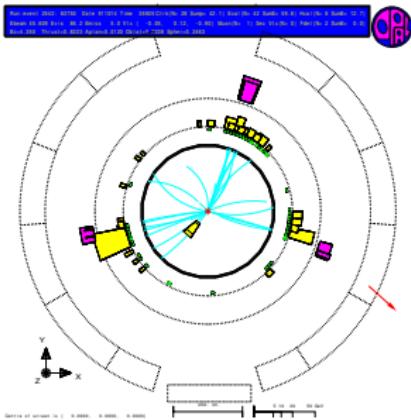
Two Soft Jet Contribution



$$L = \frac{\alpha_s C_A}{\pi} \ln \frac{\tau_L}{\tau_R}$$

The Soft Substructure of Jets

- How does an event at high energy transition from interacting partons to strongly bound non-interacting hadrons?
- How does QCD quantum mechanically transfer momentum (spin, charge, flavor, ...), between the ultra-violet and the infra-red?



The Soft Substructure of Jets

Intuition: Cascade into asymptotic *jet states* composed of soft and collinear emissions.

- Careful construction of substructure observables organizes the Cascade.
- Their factorization *precisely* and *quantitatively* realizes this intuition.
- Constrained by testable predictions.
- Enables maximal yield from modern accelerator data.

Future Directions

- N^3LL Resummation for Transverse Momentum Spectra and Precision TMD-physics.
- Calculating Factorization Violation and Multiple Parton Scattering in Hadronic Initial States.
- QCD Cascade Evolution Equations and Exclusive Factorization Theorems.
- Soft Expansion of Scattering Amplitudes.
- Next-To-Leading Power Resummation For Jet Observables.